# Analytical Analysis of TCP Performance over Geostationary Satellite Channel

Igor Bisio, *Student Member, IEEE*, Mario Marchese, *Senior Member, IEEE*

DIST - Department of Communication, Computer and System Science
University of Genoa, Via Opera Pia 13, 16145, Genoa, Italy
{ igor, dama }@dist.unige.it

*Abstract* — **The paper contains the formulation of an analytical expression of the TCP throughput and the packet loss probability over geostationary satellite channels on the basis of the state-of-the-art in the field. This work is based on the most important literature about TCP modeling and analysis. The expression obtained, in closed form, is a function of the bandwidth and buffer queues size available at the satellite gateway. Performance analysis is provided in terms of throughput and packet loss probability. Analytical results are compared with the tests obtained through a satellite network emulator already validated in the literature.**

*Index Terms* — **System performance, congestion control, satellite communication, transport protocols.**

## I. Introduction

THE Internet is the most rapidly spreading technology and many new applications find their way through the Internet.

On the other hand, satellite networks continue to be an essential element in the establishment of long distance communications and have a major role in the implementation of the so called global information infrastructures. Therefore, it is not surprising that satellite networks make Internet-based applications their primary services. In satellite networks, as well as in traditional terrestrial wired or wireless networks, the major issue that must be addressed carefully is the Quality of Service (QoS) and how the networks can provide it. In this view, one of the essential issues in designing and configuring a satellite communication network is guaranteeing QoS for different users. This action needs advanced functions to model the network status with the aim of determining and distribute the required resources.

Concerning TCP modeling, there are interesting results in the literature concerning the average congestion window size of a TCP connection [1, 2, 3], as a function of the packet loss probability, but, at best of authors' knowledge, no analytical expression of the TCP packet loss probability, in closed formula as function of the bandwidth, is available for a geostationary satellite environment.

This work proposes an expression of the TCP throughput and packet loss probability in dependence of the round trip time for each TCP connection and of the bandwidth available on the channel. The suitability of the models is evaluated through a satellite hardware emulator.

The paper is structured as follows: section II defines the general framework and summarizes the state-of-the-art used within this work. In section III a simple extension of a known TCP congestion control model is proposed. Section IV contains the formulation of the TCP packet loss. The performance evaluation is reported in Section V. Section VI lists the conclusions.

## II. Modeling TCP: Hypothesizes and Implications

The analytical expressions of the throughput and the packet loss probability when several TCP sources equally share a geostationary satellite channel is the object of this study.

### A. Scenario

The scenario considered is shown in Fig. 1. $T_n$ is the round trip time at the TCP layer for the $n$-th connection. It is supposed constant for each packet of the $n$-th connection.

$W^{pipe}$ is the maximum volume of information that can be transmitted to the system composed of a channel server of capacity $C$ and of an IP buffer of size $Q$.

Defining $C_n$ and $Q_n$, constant over time, respectively, the maximum portion of the capacity $C$ and of the buffer $Q$, "seen" by the $n$-th connection, and $W_n^{pipe}$ the maximum volume of information that can be transmitted to the system by the $n$-th connection, it is true that:

$$W^{pipe} = \sum_{j=1}^{N} W_j^{pipe} = \sum_{j=1}^{N} (C_j \cdot T_j + Q_j) \qquad (1)$$

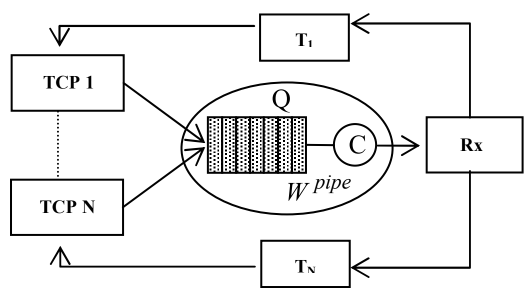


Fig. 1. TCP model.

### B. Hypothesizes

The following starting-points, called "long term behavior hypothesizes", are strictly necessary for this work:

1. each source has always data to send [3];
2. the number of sources is such to saturate the channel [3]; this hypothesis implies 1;
3. the sources are synchronized [1, 3, 4];
4. only congestion avoidance phase is considered [3];
5. the evolution of the current congestion window for a generic $i$-th connection is described by a Markov regenerative process with rewards [3];
6. losses are due only to congestion [1, 3] (i.e., only the buffer shown in Fig. 1 can lose packets);

7. the geostationary satellite is considered only a bent pipe channel, thus no on board processing is considered.

### C. *Implications from the hypothesizes*

– The flow synchronization hypothesis (3, in the previous list), extended to an aggregate of TCP sources, implies (2), where $W_j$ , $W_n$ , $T_j$ , $T_n$ are, respectively, the current congestion window (which, during the congestion avoidance phase, is the average number of packets in flight) and the round trip time for two generic sources *j* and *n* belonging to the set of integers [1, N]:

$$W_j T_j = W_n T_n , \forall j, n \in [1, N] \qquad (2)$$

– Being in the congestion avoidance phase (supposing one packet lost a time), which is the hypothesis 4, and taking TCP Reno as reference, the dimension of the congestion window $W_n$ of a generic source *n* varies between a minimum and a maximum value as introduced in [1] (TCP-Reno simplified model) and as reported in formula (3). Its size grows up to saturate the channel; if a packet is lost, the window decreases its maximum size in dependence of a factor *m* [5] that varies between 0 and 1 (typically *m*=1/2, as indicated in [1]). The receiving window is supposed not to be a congestion element and it does not play any role.

$$mW_n^{\max} \le W_n(t) \le W_n^{\max} \qquad (3)$$

The congestion window mean value $E\{W_n\}$ for the *n*-th source may be calculated by considering the dynamic of the congestion window as a function of period TD. TD is the time between two packet loss events during the congestion advoidance phase. A loss is detected by the Triple Duplicate mechanism. Being the congestion window a time function that grows linearly in the triple duplicate period, its behavior is approximated with the periodic function in the domain [0, ∞]. The initial phases of the TCP congestion control and the time out events are not considered, coherently with the hypotheses formulated above.

$$W_n(t) = \sum_{k=0}^{+\infty} \left[ \frac{1-m}{TD} W_n^{\max}(t - kTD) + mW_n^{\max} \right] \cdot \Pi\left( \frac{t-(k+1/2)TD}{TD} \right) \qquad (4)$$

Where $\Pi(\cdot)$ is a rectangular function, symmetric with respect to "*t*=0" of duration TD, shifted of TD/2 and periodized.

If the function in (4) is supposed to be a realization of the stochastic process $W_n(t)$, the mean value over a period T is:

$$\begin{aligned} \langle W_n(t) \rangle &= \frac{1}{T}\int_0^T W_n(t)dt \\ &= \frac{1}{TD}\int_0^{TD} \left( \frac{1-m}{TD} W_n^{\max}(t) + mW_n^{\max} \right) dt \\ &= \frac{m+1}{2} W_n^{\max} \end{aligned} \qquad (5)$$

Considering $W_n(t)$ ergodic, the value in equation (5) can be extended for all the possible realizations and the congestion window mean value $E\{W_n\}$ for the *n*-th source may be approximated by the intermediate value obtained in (5). $W_n^{\max}$ is maximum amount of data that can be sent by the *n*-th source. It is limited by the quantity $W_n^{pipe}$, defined in (1) hence:

$$W_n^{max} = W_n^{pipe} \qquad (6)$$

## III. THE CONGESTION WINDOW MODEL: A SIMPLE REVISION

In this section is proposed a short revision of a part of the model proposed in [3]. The reduction parameter m of the congestion window in case of loss event during the congestion avoidance phase is introduced. This choice is aimed at obtaining a complete formulation of the model proposed in this work, and at getting a closed formula of the packet loss probability and of the throughput as a function of the satellite available bandwidth, buffer sizes and protocol parameter of the TCP.

Coherently with [3], the following definition for the *n*-th TCP source are given:

- $TD_n^j$ (Triple Duplicate) is the *j*-th triple duplicate period;
- $x_n^j = TD_n^j / T_n$ is the number of rounds (which is the number round trip times $T_n$ contained in a TD period);
- $W_n^j$ is the instantaneous windows size at the end of the *j*-th, period (expressed in packets coherently with [3]);
- $W_n^{j-1}$ is the instantaneous windows size at the beginning of the *j*-th period (expressed in packets coherently with [3]);
- $b_n$ is the number of packets covered by one acknowledgement;

When a packet is lost the TCP window is reduced, as said above, *m* times [5]. Assuming $mW_n^{j-1}$ and $x_n^j / b_n$ integers.

$$W_n^j = mW_n^{j-1} + \frac{x_n^j}{b_n} \qquad (7)$$

$Y_n^j$ is the number of packets transmitted in a period of duration $A_n^j$ ([3]). The $Y_n^j$ value is expressed in equation (8), where $\beta_n^j$ is the number of packets sent in the last round before the loss event considered uniformly distributed between 1 and $W_n^j$:

$$
\begin{aligned}
Y_n^j &= \sum_{i=0}^{\frac{x_n^j}{b_n}-1} \left(mW_n^{j-1}+i\right)b_n + \beta_n^j \\
&= x_n^j \left[ mW_n^{j-1} + \frac{1}{2}\left(\frac{x_n^j}{b_n}-1\right)\right] + \beta_n^j
\end{aligned} \tag{8}
$$

Substituting equation (7) in (8),

$$
Y_n^j = \frac{x_n^j}{2}\left[mW_n^{j-1} + W_n^j - 1\right] + \beta_n^j \tag{9}
$$

Coherently with [3], to find a simple approximate solution, $W_n^j$ and $x_n^j$ are both assumed mutually independent sequences of i.i.d. random variables and, in particular, $W_n^j$ is supposed to be a Markov Regenerative process with Rewards $Y_n^j$. From equation (7), an expression of the congestion window mean value can be obtained:

$$
E\{W_n\} = \frac{E[x_n]}{b_n(1-m)} \tag{10}
$$

Reference [4] contains the formulation of $E\{Y_n\}$, the mean value of the process $Y_n^j$, as a function of $p_n$:

$$
E\{Y_n\} = \frac{1-p_n}{p_n} + E\{W_n\} \tag{11}
$$

Where $p_n > 0$ is the TCP packet loss probability for the $n$-th connection. Equaling equation (11) and the mean value of the expression in (9):

$$
\frac{1-p_n}{p_n} + E\{W_n\} = \frac{E\{x_n\}}{2}\left(mE\{W_n\} + E\{W_n\} - 1\right) + E\{\beta_n\} \tag{12}
$$

Considering that $E\{\beta_n\} = mE\{W_n\}$, equations (10) and (12), it is true that:

$$
\begin{aligned}
&\frac{1-p_n}{p_n} + E\{W_n\} = \\
&= \frac{b_n(1-m)E\{W_n\}}{2}\left[(m+1)E\{W_n\} - 1\right] + mE\{W_n\}
\end{aligned} \tag{13}
$$

After simple algebraic computations:

$$
\begin{aligned}
&E^2\{W_n\} - 2\frac{\frac{b_n(1-m)}{2} - m + 1}{b_n(1-m^2)}E\{W_n\} + \\
&-2\frac{1}{b_n(1-m^2)}\frac{1-p_n}{p_n} = 0
\end{aligned} \tag{14}
$$

Solving the equation with respect to $E\{W_n\}$ the solution may be written as a function of $m$:

$$
E\{W_n\} = \sqrt{\frac{2}{b_n(1-m^2)p_n}} + o(p_n) \tag{15}
$$

If $m$ = ½, as in TCP-Reno implementations commonly used, the same result derived in [3] for small values of the packet loss probability is found.

## IV. TCP Packet Loss

This section is aimed at obtaining an expression of the packet loss probability as a function of the bandwidth available, on the basis of the state of the art reported in the previous sections. The hypothesizes expressed in the section II are still valid.

From (2) , it is also true that, for any $j$ and $n$

$$
\frac{W_j}{W_n} = \frac{T_n}{T_j}, \forall j, n \in [1, N] \tag{16}
$$

$$
W_j = \frac{T_n}{T_j}W_n, \forall j, n \in [1, N] \tag{17}
$$

Defining $R_n^{\max}$ as the maximum value for the n-th TCP flow entering the system, it is true that:

$$
\sum_{j=1}^{N} R_j^{\max} = \sum_{j=1}^{N} \frac{W_j^{pipe}}{T_j} = \sum_{j=1}^{N} \frac{C_j \cdot T_j + Q_j}{T_j} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \tag{18}
$$

From (6) and (18),

$$
\sum_{j=1}^{N} \frac{W_j^{\max}}{T_j} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \tag{19}
$$

From equation (17), applied for the maximum value,

$$
\sum_{j=1}^{N} \frac{T_n \cdot W_n^{\max}}{T_j^2} = T_n \cdot W_n^{\max} \cdot \sum_{j=1}^{N} \frac{1}{T_j^2} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \tag{20}
$$

Extracting $W_n^{\max}$ from equation (20) and using (5), the average congestion window may be written:

$$
E\{W_n\} = \frac{m+1}{2} \cdot \left( C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \bigg/ T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2} \right) \tag{22}
$$

Matching equations (15), for small $p_n$ values, and (22):

$$\sqrt{\frac{2}{b_n(1-m^2)p_n}} = \frac{m+1}{2} \cdot \left( C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \middle/ T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2} \right) \quad (23)$$

Extracting $p_n$ from (23),

$$p_n = \frac{8}{b_n(1-m^2)(m+1)^2} \left( T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2} \right)^2 \middle/ \left( C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \right)^2 \quad (24)$$

The packet loss probability $p_n$ defined in reference [3] may be due both to congestion or to channel errors. In this paper, the packet loss event is assumed to happen in the buffer modeling the channel. The model is surely suitable for losses due to congestion, also because the loss is assumed triggered only by duplicated acknowledgements, but the bandwidth restriction (i.e., the availability to serve entering packets, which generates loss) may be considered also an effect linked to channel errors (for example, channel noise may have generated an increase of FEC bits, so reducing the bandwidth available for the data and "generating" buffer overflow).

Being a GEO satellite, the round trip time may be supposed fixed and equal for all the sources (25). This equality, together with the hypothesis 3 (synchronization), gives origin to "fairness" defined in [6], which is the condition when all the connections have the equal share of the bandwidth. If the connections do not get exactly equal allocation, "fairness" may be quantified by an index that measures the "distance" from the ideal condition (again in [6]).

$$T_j = T_n = RTT, \forall j, n \in [1, N] \quad (25)$$

This hypothesis is reasonable when the channel capacities are sufficiently wide. Emulation measures show that, in a satellite channel with propagation delay equal to 260ms and with the capacities used in this work, the round trip time obtained is approximately twice the propagation delay. The queuing delay measured does not introduce substantial variations of the round trip time and may be supposed fixed over the time.

Substituting the expression (25) in (24) and remembering that $\sum_{j=1}^{N} Q_j = Q$ and $\sum_{j=1}^{N} \frac{1}{T_j^2} = N \cdot \frac{1}{RTT^2}$:

$$p_n = \frac{8N^2}{b_n \cdot \left(1-m^2\right) \cdot \left(m+1\right)^2 \cdot (C \cdot RTT + Q)^2} \quad (26)$$

By imposing the same hypothesis (25) in (22), the value of the average congestion window for the *n*-th connection is:

$$E\{W_n\} = \frac{m+1}{2} \cdot \frac{C \cdot RTT + Q}{N} \quad (27)$$

The average throughput value:

$$\overline{R}_n = \frac{E\{W_n\}}{RTT} = \frac{m+1}{2} \cdot \frac{C \cdot RTT + Q}{N \cdot RTT} \quad (28)$$

The two quantities in (27) and (28) are independent of the index *n* (i.e. they are the same for each single source, fixed the other parameters).

## V. Performance Evaluation

In this section the analytical results provided in this paper are compared with the values got through real measures obtained through a satellite hardware emulator fully validated in the literature [7]. In particular, the configuration used in the analysis is the *Dumbbell topology*, also used in [5], where N TCP flows (active sources), which transport heavy file transfer (as considered in [4]), are conveyed towards one single node served with bandwidth $C$. The node may represent the satellite earth station gateway. The TCP parameter $m$ is set to ½; the buffer size $Q$ to 10 packets of 1500 bytes. The bandwidth $C$ is varied as well as the number of sources N. The round trip time RTT is set to 520 ms, as common for geostationary satellites.

It seems important to note that the analytical values (shown in the following figures by using white histograms) are very similar to the emulation measures (shown with gray histograms). Approximations and hypotheses made for the model derivation give origin to a slight estimation error of the measured TCP throughput and, should be clear in the following, for the measured TCP packet loss. It happens because, in the model, the system is supposing operating in regime condition, neglecting the instantaneous dynamics of the system. Fig. 2 shows a situation where several TCP connections are conveyed through a satellite channel with a fixed capacity available (1Mb/s). Throughput values are indicated both for the emulation (Measure) and for analytical model proposed in formula (28) (Analysis). In both cases the performance in terms of throughput decreases with the same slope when the number of active TCP source increases.

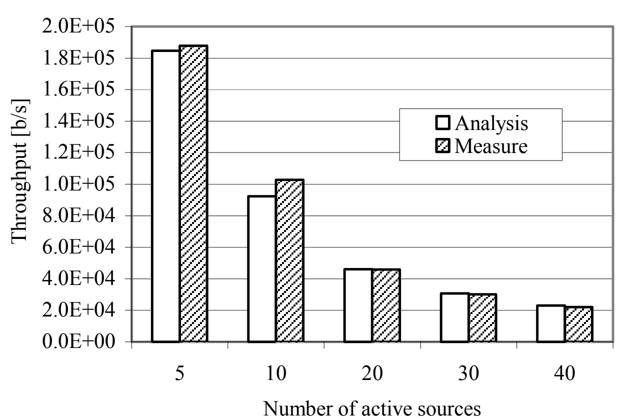


Fig. 2. Throughput comparison vs number of active sources (C=1Mb/s).

Fig. 3 contains the same quantities of Fig. 2 but the bandwidth available is 2Mb/s. The throughput of the TCP connections is very high and decreases when the number of sources saturates the capacity of the satellite network. Also in this case the analytical analysis follows the real behavior of the measured throughput.

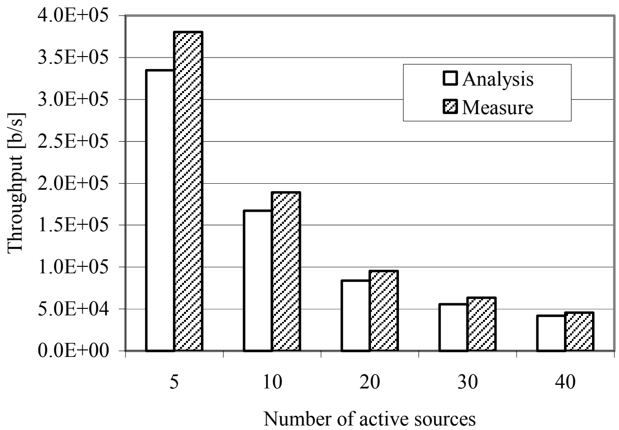


Fig. 3. Throughput comparison vs number of active sources (C=2Mb/s).

Concerning the packet loss probability (shown in Fig. 4 and Fig. 5, for bandwidth 1Mb/s and 2Mb/s, respectively), the analytical values result to be really accurate, compared with the emulated values.

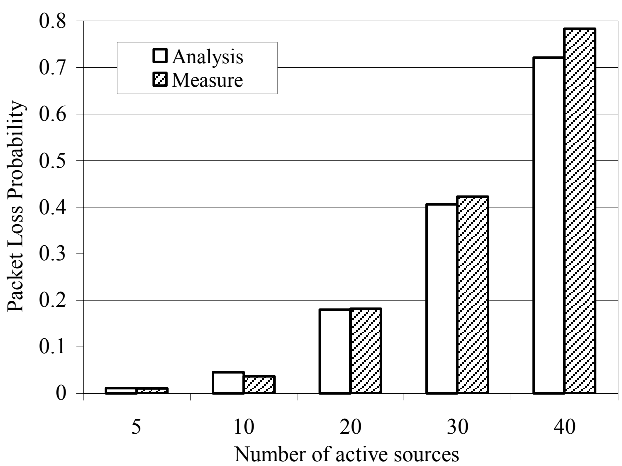


Fig. 4. Packet loss Probability vs number of active sources (C=1Mb/s).

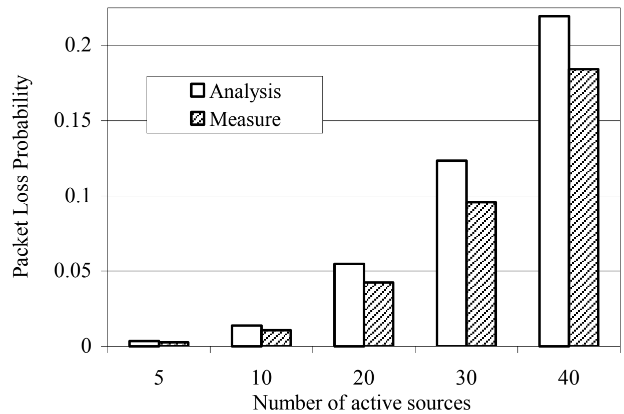


Fig. 5. Packet Loss Probability vs number of active sources (C=2Mb/s).

The measures obtained via emulator are approximately the same of the analytical values computed by using (26) and (28), and it is clear in all the figures reported.

## VI. Conclusions

An analytical expression of the TCP connection throughput and packet loss probability as a function of the bandwidth and buffer available have been introduced. The analytical results have been compared with measures obtained by using a hardware satellite emulator and show a high degree of accuracy. The model provided in this paper guarantees a reliable evaluation of the performance of the TCP congestion control behavior over geostationary satellite networks. Being in closed form, the quantities obtained may be used to optimize the TCP protocol performance within the framework of control mechanisms.

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