Analytical Expression and Performance Evaluation of TCP Packet Loss Probability Over Geostationary Satellite

Igor Bisio, *Student Member, IEEE,* and Mario Marchese, *Member, IEEE*

***Abstract*—The letter contains the formulation of an analytical expression of the Transmission Control Protocol (TCP) packet loss probability over geostationary satellite channels on the basis of the state-of-the-art in the field. The expression obtained is function of the bandwidth available and is suited to be used in control mechanisms. It is compared with the results obtained through a satellite network emulator already validated in the literature.**

***Index Terms*—Communication system performance, flow control, satellite communication, transport protocols.**

I. Introduction

CONCERNING User Datagram Protocol (UDP) traffic, [1] provides an analytical approximation of the packet loss probability, as a function of the bandwidth allocated, after fixing the traffic model, which is used in the literature to describe all the IP-based traffic with no distinction between UDP and TCP. Nevertheless, the particular type of statistical ON-OFF fractal traffic nature suggests its use for UDP traffic, where no acknowledgment-based mechanism is provided for congestion control.

Concerning TCP, there are interesting results in the literature concerning the average congestion window size of a TCP connection [2]–[4], as a function of the packet loss probability, but, at best of authors' knowledge, no analytical expression of the TCP packet loss probability, in closed formula as function of the bandwidth is provided in the literature.

This work, starting from the state-of-the-art, proposes an expression of the TCP packet loss probability in dependence of the round-trip time for each TCP connection and of the bandwidth available on the channel. The scope of the work is then restricted to geostationary (GEO) satellite environment. The efficiency of the formulation is evaluated through a satellite emulator.

This letter is structured as follows. Section II defines the framework of this research and summarizes the state-of-the-art used within this work. Section III describes the formulation of the TCP packet loss proposed in the paper. The performance evaluation is reported in Section IV. Section V contains the conclusions.

Manuscript received August 1, 2003. The associate editor coordinating the review of this letter and approving it for publication was Prof. M. Ma.

The authors are with the Italian National Consortium for Telecommunications (CNIT), 16145 Genova, Italy (e-mail: igor.bisio@cnit.it; mario.marchese@cnit.it).

Digital Object Identifier 10.1109/LCOMM.2004.825723

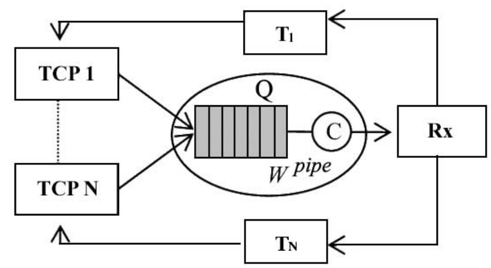

Fig. 1. TCP model.

II. Modeling TCP: Hypothesizes and Implications

A. Scenario

The scenario considered is shown in Fig. 1. T_n is the round-trip time at the TCP layer for the nth connection. It is supposed constant for each packet of the nth connection.

$W^{\rm pipe}$ is the maximum volume of information that can be transmitted to the system composed of a channel server of capacity C bit/s and of an IP buffer of size Q bit, with no loss due to congestion.

Defining C_n and Q_n, constant over time, respectively, the maximum portion of the capacity C and of the buffer Q, "seen" by the nth connection, and $W_n^{\rm pipe}$ the maximum volume of information that can be transmitted to the system by the nth connection, it is true that

$$W^{\rm pipe} = \sum_{j=1}^{N} W_j^{\rm pipe} = \sum_{j=1}^{N} (C_j \cdot T_j + Q_j) \tag{1}$$

B. Hypothesizes

The following starting-points, called "long term behavior hypothesizes", are strictly necessary for this work:

1) each source has always data to send [4];
2) the number of sources is such to saturate the channel [4]; this hypothesis implies 1;
3) the sources are synchronized [2], [4], [5];
4) only congestion avoidance phase is considered [4].
5) the evolution of the current congestion window for a generic ith connection is described by a Markov regenerative process with rewards [4];
6) losses are due only to congestion [2], [4].

C. Implications

— The flow synchronization hypothesis (3, in the previous list), extended to an aggregate of TCP sources, implies (2), where W_j, W_n, T_j, T_n are, respectively, the current congestion window (which, during the congestion avoidance phase, is the average number of packets in flight) and the round-trip time for two generic sources j and n belonging to the set of integers $[1, N]$:

$$W_j T_j = W_n T_n \ \forall j, n \in [1, N] \quad (2)$$

— Being in the congestion avoidance phase (supposing one packet lost a time), which is the hypothesis 4, and taking TCP Reno as reference, the dimension of the congestion window W_n of a generic source n varies between a minimum and a maximum value as introduced in [2] (TCP-Reno simplified model) and as reported in (3). Its size grows up to saturate the channel; if a packet is lost, the window decreases its maximum size in dependence of a factor m [6] that varies between 0 and 1 (typically $m = 1/2$, as indicated in [2]). The receiving window is supposed not to be a congestion element and it does not play any role.

$$mW_n^{\max} \le W_n \le W_n^{\max} \quad (3)$$

The congestion window mean value $\overline{W}_n$ for the nth source may be approximated by the intermediate value obtained by (3), setting

$$\overline{W}_n = \frac{m+1}{2} W_n^{\max} \quad (4)$$

where $W_n^{\max}$ is maximum amount of data that can be sent by the nth source without receiving acknowledgment. It is limited by the quantity W_n^{pipe} defined in (1).

$$W_n^{\max} = W_n^{\text{pipe}} \quad (5)$$

— Reference [4] proposes a formulation, reported in (6), for the average congestion window of the nth TCP connection by using the hypothesis 1, 4, and 5 and referring to TCP Reno.

$$\overline{W}_n(p_n) = \sqrt{\frac{8}{3b_n p_n}} + o\left(\frac{1}{\sqrt{p_n}}\right) \quad (6)$$

p_n is the packet loss probability and b_n the number of packets covered by one acknowledgment for connection n. $o(\cdot)$ is a function of p_n, which is negligible for very small p_n values.

III. TCP Packet Loss

This section is aimed at obtaining a novel expression of the packet loss probability as a function of the bandwidth available, on the basis of the state of the art reported in the previous section. The hypothesizes expressed in the previous section are still valid.

From (2), it is also true that, for any j and n

$$\frac{W_j}{W_n} = \frac{T_n}{T_j} \ \forall j, n \in [1, N] \quad (7)$$

$$W_j = \frac{T_n}{T_j} W_n \ \forall j, n \in [1, N] \quad (8)$$

Defining R_n^{max} as maximum value for the nth TCP flow entering the system, it is true that

$$\sum_{j=1}^{N} R_j^{max} = \sum_{j=1}^{N} \frac{W_j^{\text{pipe}}}{T_j} = \sum_{j=1}^{N} \frac{C_j \cdot T_j + Q_j}{T_j} = \sum_{j=1}^{N} C_j + \sum_{j=1}^{N} \frac{Q_j}{T_j} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \quad (9)$$

From (5) and (9)

$$\sum_{j=1}^{N} \frac{W_j^{\max}}{T_j} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \quad (10)$$

From (8), applied for the maximum value

$$\sum_{j=1}^{N} \frac{T_n \cdot W_n^{\max}}{T_j^2} = T_n \cdot W_n^{\max} \cdot \sum_{j=1}^{N} \frac{1}{T_j^2} = C + \sum_{j=1}^{N} \frac{Q_j}{T_j} \quad (11)$$

and

$$W_n^{\max} = \frac{C + \sum_{j=1}^{N} \frac{Q_j}{T_j}}{T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2}}. \quad (12)$$

From (4)

$$\overline{W}_n = \frac{m+1}{2} \cdot \frac{C + \sum_{j=1}^{N} \frac{Q_j}{T_j}}{T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2}}. \quad (13)$$

Matching (6), which is true with $m = 1/2$, for small p_n values, and (13)

$$\sqrt{\frac{8}{3b_n p_n}} = \frac{m+1}{2} \cdot \frac{C + \sum_{j=1}^{N} \frac{Q_j}{T_j}}{T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2}}. \quad (14)$$

Extracting p_n from (14) and assuming $m = 1/2$ in the remainder of the letter

$$p_n = \frac{32}{3b_n \cdot (m+1)^2} \frac{\left(T_n \cdot \sum_{j=1}^{N} \frac{1}{T_j^2}\right)^2}{\left(C + \sum_{j=1}^{N} \frac{Q_j}{T_j}\right)^2}. \quad (15)$$

Being a GEO satellite, the round-trip time may be supposed fixed and equal for all the sources (16). This equality, together with the hypothesis 3 (synchronization), gives origin to "fairness" (checked in Section IV), which is the condition when all the connections have the equal share of the bandwidth [7]. If the connections do not get exactly equal allocation, "fairness" may be quantified by an index that measure the "distance" from the ideal condition [7].

$$T_j = T_n = \text{RTT} \ \forall j, n \in [1, N]. \quad (16)$$

Substituting (16) in (15) and remembering that $\sum_{j=1}^{N} Q_j = Q$, being Q the overall buffer size, and that $\sum_{j=1}^{N}(1/T_j^2) = N \cdot (1/\text{RTT}^2)$, the following equation in (17) is obtained:

$$p_n = \frac{32N^2}{3b_n \cdot (m+1)^2 \cdot (C \cdot \text{RTT} + Q)^2} \quad (17)$$

By imposing the same hypothesis (16) in (13), the value of the average congestion window for the nth connection is

$$\overline{W}_n = \frac{m+1}{2} \cdot \frac{C \cdot \text{RTT} + Q}{N} \quad (18)$$

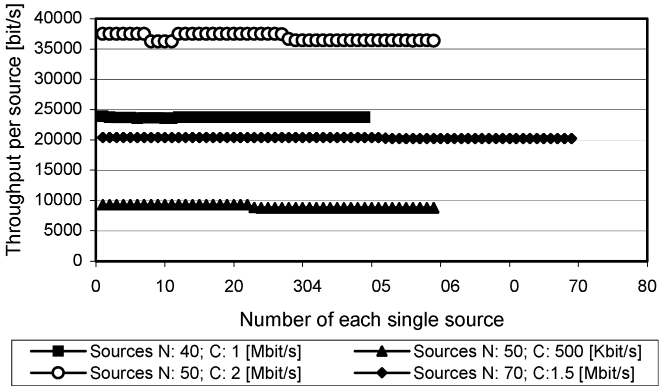

Fig. 2. Fairness behavior in satellite channel.

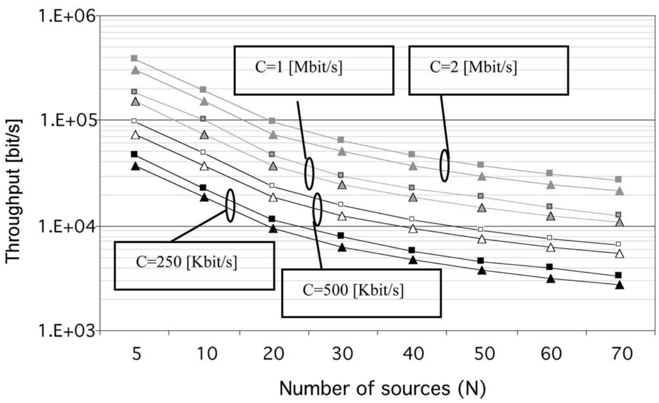

Fig. 3. Throughput versus number of sources.

and the average throughput value is

$$\overline{R}_n = \frac{\overline{W}_n}{\mathrm{RTT}} = \frac{m+1}{2} \cdot \frac{C \cdot \mathrm{RTT} + Q}{N \cdot \mathrm{RTT}} \tag{19}$$

The two quantities in (18) and (19) are independent of the index n (i.e., they are the same for each single source, fixed the other parameters).

IV. Performance Evaluation

The first step is aimed at checking the fairness behavior hypothesis made to get the final throughput in (19) and the packet loss in closed formula of (17). Fig. 2 contains the tests obtained through a satellite hardware emulator fully validated in the literature [8]. The tests have been carried out by varying the overall number N of sources and the capacity C. The buffer length Q is fixed to 160 Kbits, the value m to 1/2, the round-trip time, RTT, to 520 ms, as common for geostationary satellites. The bit error rate over the channel is supposed negligible: all losses are due to congestion. The throughput value, measured for each single source, is almost the same for each single source, fixed the maximum number of sources, the capacity and the buffer size. The system is operating fairly. The measures obtained via emulator are approximately the same of the analytical values that can be computed from (19). This last concept is explicitly clarified in Fig. 3, where the throughput values in bits per second (bits/s) obtained via emulation are directly compared with the analytical values computed through (19). The configuration used in the test is the *Dumbbell topology* [6], where N TCP flows are conveyed toward one single node served with bandwidth C. Parameter m is set again to 1/2. The buffer size Q to 160 kbits. The bandwidth C is varied as well as the number of sources N. The analytical values (shown by using triangles) are almost overlapped to the emulation measures (shown with squares).

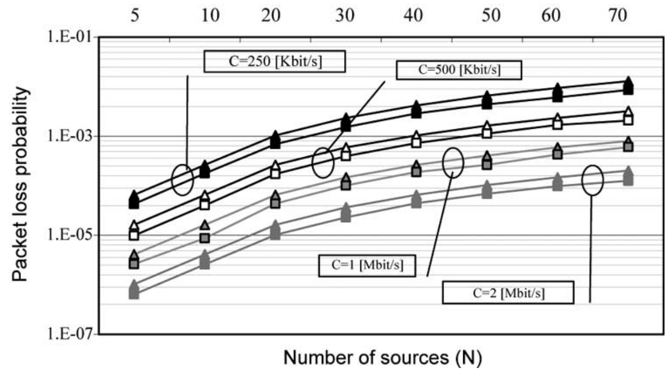

Fig. 4. Packet loss probability versus number of sources.

The packet loss probability in (17) is tested in Fig. 4, where the values obtained analytically (shown with triangles) are compared with the emulation measures (reported with squares), by applying the same definition of parameters used for Fig. 3. Also in this case the analytical values are approximately the same of the emulation measures.

V. Conclusions

An analytical expression of the TCP connection packet loss probability as a function of the bandwidth available has been introduced (as well as the computation of the throughput) for GEO satellites. The analytical results (compared with measures obtained by a hardware satellite emulator) have shown a high degree of accuracy. The quantities obtained may be used to design control mechanisms and to provide performance analysis.

References

[1] B. Tsybakov and N. D. Georganas, “On self-similar traffic in ATM queues: definition, overflow probability bound, and cell delay distribution,” *IEEE/ACM Trans. Networking*, vol. 5, pp. 397–409, June 1997.

[2] T. Lakshman and U. Madhow, “The performance of TCP/IP for networks with high bandwidth-delay products and random loss,” *IEEE/ACM Trans. Networking*, vol. 5, pp. 336–350, June 1997.

[3] E. Altman, K. Avrachenkov, and C. Barakat, “A stochastic model of TCP/IP with stationary random loss,” in *Proc. ACM SIGCOMM*, Stockholm, Sweden, 2000, pp. 231–242.

[4] J. Padhye, V. Firoiu, D. Towsley, and J. Kurose, “Modeling TCP throughput: a simple model and its empirical validation,” in *Proc. ACM SIGCOMM*, Vancouver, Canada, 1998, pp. 303–314.

[5] E. Altman, C. Barakat, E. Laborde, P. Brown, and D. Collange, “Fairness analysis of TCP/IP,” in *Proc. Conf. on Decision and Control (ICDC)*, Sydney, Australia, 2000, pp. 61–66.

[6] J. P. Hespanha, S. Bohacek, K. Obraczka, and J. Lee, “Hybrid modeling of TCP congestion control,” presented at the *HSCC Int. Workshop*, Rome, Italy, 2001.

[7] D. Chiu and R. Jain, “Analysis of the increase/decrease algorithms for congestion avoidance in computer networks,” *J. Computer Networks and ISDN*, vol. 17, pp. 1–14, June 1989.

[8] F. Davoli and M. Marchese, “Satellite system simulation, techniques and applications,” in *Applied System Simulation: Methodologies and Applications*, M. Obaidat and G. I. Papadimitriou, Eds. Norwell, MA: Kluwer Academic, 2003.