A Model of Buffer Occupancy for ICNs

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***Abstract*—In this letter, an analytical framework to model nodes in Intermittently Connected Networks (ICNs) is proposed. A relationship is derived in the z-domain between the discrete probability densities of their buffer state occupancies and the sizes of the arriving bulks. Under a fixed epidemic-routing-based forwarding strategy, expressions are obtained for the average buffer occupancy and its standard deviation with immediate protocol advantages.**

***Index Terms*—Intermittently connected networks, congestion control, ad-hoc networks, Markov chains, epidemic routing.**

I. Introduction

In the last years various applications emerged, where networks operate under conditions in which the assumptions of "universal connectivity" and "global information" do not hold. Examples are sensor networks and vehicular ad-hoc networks. A common denomination of such contexts is *Intermittently Connected Networks* (ICNs). The networks may be disconnected most of the time and it may even happen that there is no end-to-end path available at the same time between a source and a destination. In such contexts, classical routing and data delivery-approaches fail [1]. In such cases, one of the most common approaches is *epidemic routing* [2], which is based on the replication and transmission of messages to newly-discovered contacts.

In this letter we propose an analytical framework, based on bulk arrival and bulk service queues, to model ICN nodes behavior (Section II). We compute the stationary discrete probability densities of the state occupancies of the ICN node buffers, in terms of the discrete probability densities of the sizes of the arriving bulks (Section III). Then, we investigate a class of forwarding strategies, based on epidemic routing, used by ICN nodes (Section IV). For this class of forwarding strategies, we derive an expression for the average buffer occupancy (Section V). Finally, we discuss related literature (Section VI) and draw some conclusions (Section VII).

II. Model Description

We consider the following ICN scenario. M nodes, deployed in an area, follow a certain mobility model, for which we impose the following property: for each couple of ICN nodes ($m, n \in \{1, \ldots, M\}, m \neq n$) the number of encounters between them in any given interval of time is a Poisson random variable. This obviously means that the inter-meeting time between two generic ICN nodes m and n is an exponentially-distributed random variable. Popular mobility models such as Random Waypoint and Random Direction [3] enjoy such a property.

We denote by $\mathcal{L} \subseteq \{1, \ldots, M\}$ the set of destination nodes. We model each node as a battery of queues (Figure 1). Within a specific node $j \in \{1, \ldots, M\}$, there are $|\mathcal{L}(j)|$ queues, where $\mathcal{L}(j) \triangleq \mathcal{L} \setminus \{j\}$. Each l-queue ($l \in \mathcal{L}(j)$) within the node j receives incoming data for the destination node l in two different modes. Either the data directed to l are internally generated by node j or they have been sent to j by other nodes during previous encounters with j, on the basis of a forwarding strategy. When node $j \in \{1, \ldots, M\}, j \neq l$ encounters node l which is the destination of data it holds in its l-queue, it empties the l-queue completely sending all its data to l. To allow this operation, we assume that the maximum data exchange time between two nodes is much smaller than the average duration of the encounter.

More formally, node j encounters the destination node l with average rate $\mu^{j,l}$ [encounters/s] and sends to l all the packets buffered in its l-queue. Node j generates data in bulks, assigned to l, with average rate $r_s^{j,l}$ [generations/s] and, at each generation, it produces $I_s^{j,l}$ [bulks/generation], set to 1 in this letter. The average rate of bulk generation is $\lambda_s^{j,l} = r_s^{j,l} I_s^{j,l}$ [bulks/s]. We assume an exponentially-distributed time between two consecutive bulk generations. Node j meets any node different from the destination l with average rate $E_e^{j,l} = \sum_{h=1, h\neq j,l}^{M} \mu^{j,h}$ [encounters/s] and, at each encounter, receives $I_e^{j,l}$ [bulks/encounter], set to 1 in this letter. The corresponding bulk generation process has rate $\lambda_e^{j,l} = E_e^{j,l} I_e^{j,l}$ [bulks/s] and is a Poisson process, since it is the sum of independent Poisson processes. The two processes of bulk generation with associated average rates $\lambda_s^{j,l}$ and $\lambda_e^{j,l}$ are assumed to be independent. Due to the assumption on the mobility model and on the generation of bulks, the global process of bulk arrivals in the l-queue is a Poisson process. We denote by $\lambda^{j,l} = \lambda_s^{j,l} + \lambda_e^{j,l}$ [bulks/s] its average rate.

The size of each bulk (i.e., the number of packets in the bulk) is also a random variable. We denote by $g_{k,e}^{j,l}$ the probability that the bulk assigned to l and received by j during an encounter is composed of k packets, and by $g_{k,s}^{j,l}$ the probability that the bulk generated by node j and assigned to node l is composed of k packets. The average arrival rate of bulks of k packets in the l-queue, measured in [packets/s], is $\lambda_s^{j,l} g_{k,s}^{j,l} + \lambda_e^{j,l} g_{k,e}^{j,l}$, as indicated in Figure 1. For $k \in \mathbb{N}_0$, we denote by $\{g_{k,s}^{j,l}\}$ and $\{g_{k,e}^{j,l}\}$ the sequences whose components are $g_{k,s}^{j,l}$ and $g_{k,e}^{j,l}$, respectively; $\{g_{k,s}^{j,l}\}$ and $\{g_{k,e}^{j,l}\}$ represent the discrete probability densities of the sizes of the two kinds of

Manuscript received February 22, 2012. The associate editor coordinating the review of this letter and approving it for publication was Y.-D. Lin.

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Digital Object Identifier 10.1109/LCOMM.2012.042312.120369

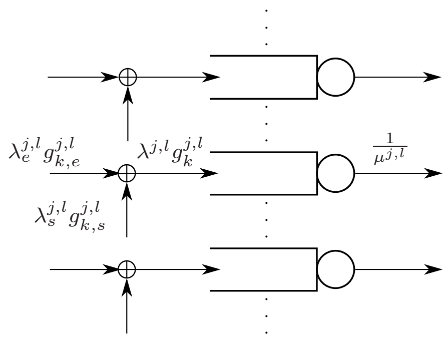

Fig. 1. Model of the generic ICN node $j \in \{1, \dots, M\}$ and its l-queue, where $l \in \mathcal{L}(j)$ ($k \in \mathbb{N}_0$).

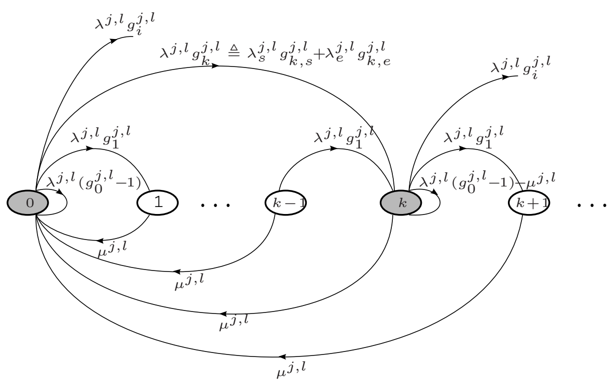

Fig. 2. Transition rates for the continuous-time Markov chain related to l-queue inside node j.

bulks. We denote by $p_k^{j,l}$ the stationary probability that the l-queue of node j has k packets and by $\{p_k^{j,l}\}$ the sequence that represents the discrete probability density of the size of the l-queue in node j.

The model introduced above allows us to model the evolution of each l-queue as a continuous-time Markov chain with bulk arrivals and bulk services. The transition rate from a generic state h to the state $h+k$ is $A_k^{j,l} = \lambda_s^{j,l} g_{k,s}^{j,l} + \lambda_e^{j,l} g_{k,e}^{j,l}$, which is the average arrival rate of bulks of length k. On the other hand, if we consider the overall bulk arrival process with average rate $\lambda^{j,l} = \lambda_s^{j,l} + \lambda_e^{j,l}$, then $A_k^{j,l}$ can be expressed as $A_k^{j,l} = \lambda^{j,l} g_k^{j,l}$ where $g_k^{j,l}$ is the probability of a k-length arrival. So, $g_k^{j,l}$ can be simply computed as in (1):

$$g_k^{j,l} = \frac{\lambda_s^{j,l}}{\lambda_s^{j,l} + \lambda_e^{j,l}} g_{k,s}^{j,l} + \frac{\lambda_e^{j,l}}{\lambda_s^{j,l} + \lambda_e^{j,l}} g_{k,e}^{j,l} \,. \tag{1}$$

The quantity $g_{k,s}^{j,l}$ can be interpreted as an endogenous component, since it is associated with the packets generated inside the currently considered node, and $g_{k,e}^{j,l}$ as an exogenous component, since, in general, it depends on the forwarding strategies of the other nodes (see Section IV for a class of possible models for $\{g_{k,e}^{j,l}\}$). The two terms $\frac{\lambda_s^{j,l}}{\lambda_s^{j,l}+\lambda_e^{j,l}}$ and $\frac{\lambda_e^{j,l}}{\lambda_s^{j,l}+\lambda_e^{j,l}}$ in (1) play the role of weights. We denote by $\{g_k^{j,l}\}$ the sequence composed by the $g_k^{j,l}$ values, which represents the discrete probability density of the size of the bulk (independently from its origin). The complete model of the Markov chain with its transition rates is shown in Figure 2.

To simplify the notation, supposing to refer to node j and queue l, in the remainder of the letter we omit the superscripts j and l. So, (1) becomes

$$g_k = \frac{\lambda_s}{\lambda_s + \lambda_e} g_{k,s} + \frac{\lambda_e}{\lambda_s + \lambda_e} g_{k,e} \,. \tag{2}$$

III. A Relationship Between Arriving Bulk Size and Queue Size

The behavior of the Markov chain introduced in Section II (which is represented by $\{p_k\}$) depends on the discrete probability density $\{g_k\}$ of the arriving bulk size in the analyzed queue. The next proposition provides a relationship between $\{p_k\}$ and $\{g_k\}$ by using their z-transforms $P(z) \triangleq \sum_{k=0}^{\infty} p_k z^{-k}$ and $G(z) \triangleq \sum_{k=0}^{\infty} g_k z^{-k}$.

Proposition 3.1: $P(z)$ and $G(z)$ satisfy

$$P(z) = \frac{\mu}{(\lambda + \mu) - \lambda G(z)} \,. \tag{3}$$

Proof: By inspection of the transition rates shown in Figure 2, we can write down the following equilibrium equations:

$$\sum_{i=1}^{\infty} p_i \mu = \sum_{i=1}^{\infty} p_k \lambda g_i \,, \qquad k = 0 \,, \tag{4}$$

$$\sum_{i=0}^{k-1} p_i \lambda g_{k-i} = \sum_{i=1}^{\infty} p_k \lambda g_i + p_k \mu \,, \qquad k \geq 1 \,. \tag{5}$$

As $\sum_{i=1}^{\infty} p_i = 1 - p_0$ and $\sum_{i=0}^{\infty} g_i = 1$, formulas (4) and (5) become

$$\mu(1-p_0) = p_0 \lambda (1-g_0) \Rightarrow p_0 = \frac{\mu}{\lambda + \mu - \lambda g_0} \,, \tag{6}$$

$$\sum_{i=0}^{k-1} p_i \lambda g_{k-i} = p_k \lambda (1-g_0) + p_k \mu = (\lambda(1-g_0) + \mu) p_k . \tag{7}$$

Following an approach similar to the one used in [4], by computing the z-transforms of both sides of (7) we get

$$\sum_{k=1}^{\infty} \sum_{i=0}^{k-1} p_i \lambda g_{k-i} z^{-k} = (\lambda(1-g_0) + \mu) \sum_{k=1}^{\infty} p_k z^{-k} \,. \tag{8}$$

Proceeding likewise in [4, pp.134-139], we interchange the summations and reorder the terms, thus getting

$$\begin{aligned} \sum_{k=1}^{\infty} \sum_{i=0}^{k-1} p_i \lambda g_{k-i} z^{-k} &= \lambda \sum_{i=0}^{\infty} p_i z^{-i} \sum_{k=i+1}^{\infty} g_{k-i} z^{-(k-i)} \\ &= \lambda \sum_{i=0}^{\infty} p_i z^{-i} \sum_{j=1}^{\infty} g_j z^{-j} \,. \end{aligned} \tag{9}$$

By (8) and (9) we obtain

$$\lambda \sum_{i=0}^{\infty} p_i z^{-i} \sum_{j=1}^{\infty} g_j z^{-j} = (\lambda(1-g_0) + \mu) \sum_{k=1}^{\infty} p_k z^{-k} \,. \tag{10}$$

In terms of z-transforms, formula (10) gives

$$\lambda P(z)(G(z) - g_0) = (\lambda + \mu - \lambda g_0)(P(z) - p_0) \,. \tag{11}$$

Finally, by substituting $p_0 = \frac{\mu}{\lambda+\mu-\lambda g_0}$ in (11), we get (3). ∎

IV. Forwarding Strategies

In this section and in the next one, we suppose that all the nodes of the network have the same traffic parameters, follow the same forwarding strategy, and, by symmetry considerations, have the same stationary discrete probability density $\{p_k\}$. Depending on the forwarding strategy used by the nodes, the discrete probability density $\{g_{k,e}\}$ for each node may depend on the state of the encountered node. Under stationary conditions and taking into account that $\{p_k\}$ does not depend on the node, this means that $\{g_{k,e}\}$ may depend on $\{p_k\}$ (see (12) for a possible dependence).

A possible forwarding strategy, which is a way to implement epidemic routing, is the following: when a node meets another one that is different from the destination, the latter exchanges the entire content of its buffer with probability q, otherwise no exchange is performed with probability $(1-q)$. In other words, suppose that the node analyzed j, which contains the l-queue under study, encounters the node $i \neq l$. Node i downloads to j all the content of its l-buffer with probability q. The size of such an l-buffer is ruled, as said above, by $\{p_k\}$, for any node. So, the size of the l-buffer in the node i is k with probability p_k, and i sends k packets to j with probability $p_k q$. On the other hand, node i does not send anything to node j in two cases: the first one happens with probability $p_0 q$, i.e., when the buffer in i used for l is empty; the second one happens with probability $(1-q)$ because of the forwarding strategy. More formally, if we denote by $\{\delta_{k,h}\}$ the Kronecker delta ($\delta_{k,h} \triangleq 1$ for $k=h$ and $\delta_{k,h} \triangleq 0$ otherwise), $\{g_{k,e}\}$ is given by

$$\{g_{k,e}\} = (1-q)\{\delta_{k,0}\} + q\{p_k\}\,. \tag{12}$$

Let $G_s(z)$ and $G_e(z)$ be the z-transforms of $\{g_{k,s}\}$ and $\{g_{k,e}\}$, respectively. Proposition 4.1 provides an expression for the z-transform $P(z)$ of the sequence $\{p_k\}$ under the class of forwarding strategies (12). The assumption $q < \mu/\lambda_e$ in Proposition 4.1 is needed for the finiteness of the average occupancy of the buffer, as it follows from the next formulas (17) and (21).

Proposition 4.1: If $\{g_{k,e}\}$ has the form (12), then

$$P(z) = \frac{\mu}{(\lambda+\mu) - (\lambda_s G_s(z) + \lambda_e)} \tag{13}$$

for $q = 0$, and

$$P(z) = \frac{\lambda+\mu-\lambda_s G_s(z) - \lambda_e(1-q)}{2\lambda_e q} - \frac{\sqrt{(\lambda+\mu-\lambda_s G_s(z)-\lambda_e(1-q))^2 - 4\lambda_e \mu q}}{2\lambda_e q} \tag{14}$$

for $0 < q \leq 1$ and $q < \mu/\lambda_e$.

Proof: By (12) and (2) we get

$$G(z) = \frac{\lambda_s}{\lambda_s+\lambda_e} G_s(z) + \frac{\lambda_e}{\lambda_s+\lambda_e}[(1-q)+qP(z)]\,. \tag{15}$$

Then for $q=0$ the statement follows by Proposition 3.1, taking into account that $\lambda = \lambda_s + \lambda_e$. Proceeding in a similar way, for the case $0 < q \leq 1$ and $q < \mu/\lambda$ one obtains a second-order algebraic equation for $P(z)$ of the form

$$\alpha P^2(z) + \beta(z) P(z) + \gamma = 0\,, \tag{16}$$

where $\alpha \triangleq \lambda_e q$, $\beta(z) \triangleq -[\lambda+\mu-\lambda_s G_s(z) - \lambda_e(1-q)]$, and $\gamma \triangleq \mu$. Finally, between the two solutions $P(z) = \frac{-\beta(z)\pm\sqrt{\beta^2(z)-4\alpha\gamma}}{2\alpha}$ of (16), we take $P(z) = \frac{-\beta(z)-\sqrt{\beta^2(z)-4\alpha\gamma}}{2\alpha}$ since, for $q < \mu/\lambda_e$, is the only one compatible with the constraint $P(1) = G_s(1) = 1$, mandatory because $\{p_k\}$ and $\{g_{k,s}\}$ are discrete probability densities. ■

For the following analysis, the values of $P'(z)$ and $P''(z)$ (the first and the second complex derivatives of $P(z)$, respectively) computed at $z=1$ are also needed. Starting from the expression of $P(z)$ in Proposition 4.1, simple computations provide the following corollary.

Corollary 4.2: If $\{g_{k,e}\}$ has the form (12), then

$$P'(1) = \frac{\lambda_s}{\mu} G'_s(1)\,, \tag{17}$$

$$P''(1) = \frac{\lambda_s}{\mu} G''_s(1) + 2\frac{\lambda_s^2}{\mu^2}(G'_s(1))^2\,, \tag{18}$$

for $q=0$, and

$$P'(1) = \frac{\lambda_s}{\mu - \lambda_e q} G'_s(1)\,, \tag{19}$$

$$P''(1) = \frac{\lambda_s}{\mu-\lambda_e q} G''_s(1) + \frac{2\lambda_s^2 \mu}{(\mu-\lambda_e q)^3}(G'_s(1))^2\,, \tag{20}$$

for $0 < q \leq 1$ and $q < \mu/\lambda_e$.

Note that the computations of formulas (17)-(20) do not require inverting z-transforms. Note also from (17)-(20) that, for what concerns the computation of the first and second order derivatives of $P(z)$ in $z=1$, (13) can be interpreted as the limit case of (14) for $q \to 0^+$.

V. Buffer Occupancy

The analysis detailed in the previous sections allows us to analyze the average buffer occupancy (21) and its standard deviation (22). The next equation is obtained starting from the definition of the z-transform: $P(z) \triangleq \sum_{k=0}^{\infty} p_k z^{-k}$, performing the derivative $P'(z) = -\sum_{k=0}^{\infty} k p_k z^{-(k+1)}$, and imposing $z=1$:

$$\sum_{i=0}^{\infty} i p_i = -P'(1)\,. \tag{21}$$

Similarly we obtain:

$$\sqrt{\sum_{i=0}^{\infty}\left(i - \sum_{k=0}^{\infty} k p_k\right)^2 p_i} = \sqrt{P''(1)(P''(1) - 2P'(1))}\,. \tag{22}$$

Formulas (21) and (22) are checked by exchanging the order of differentiation and summation in the definitions of $P'(z)$ and $P''(z)$, then taking $z=1$. Figure 3 shows the behaviors of the average buffer occupancy (21) and its standard deviation (22) for the class of forwarding strategies (12), by varying the parameter q and the values $\mu, \lambda_e, \lambda_s$. For illustrative purposes, we consider for the endogenous component $\{g_{k,s}\}$ a model in which the conditions $G'_s(1) = -1$ and $G''_s(1) = 2$ hold. An example of such a model is

$$\{g_{k,s}\} = \{\delta_{k,1}\}\,, \tag{23}$$

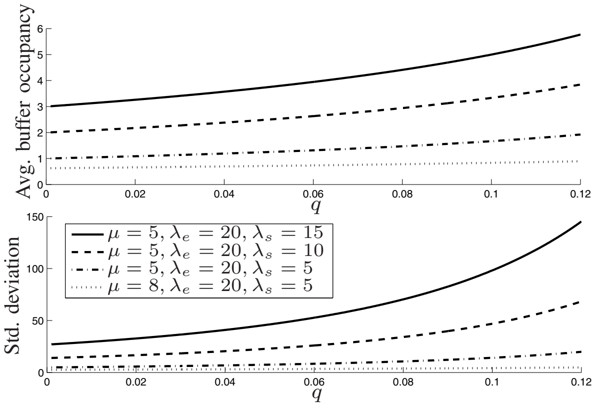

Fig. 3. Average buffer occupancy and its standard deviation for the class of forwarding strategies (12), by varying the parameter q and considering a model for the endogenous component $\{g_{k,s}\}$ for which one has $G'_s(1) = -1$ and $G''_s(1) = 2$.

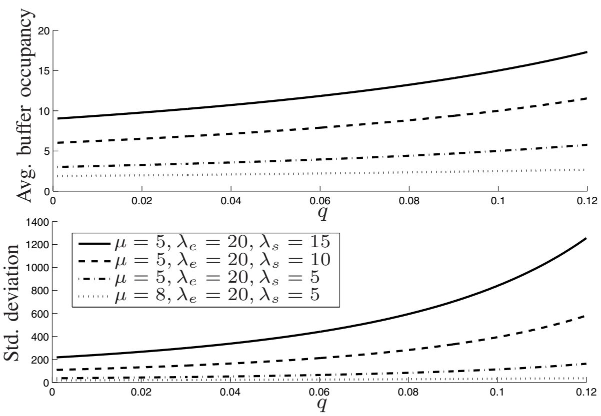

Fig. 4. Average buffer occupancy and its standard deviation with a Poisson discrete probability density for the endogenous component $\{g_{k,s}\}$ for which $G'_s(1) = -3$ and $G''_s(1) = 15$.

whose z-transform is

$$G_s(z) = z^{-1}\,, \tag{24}$$

i.e. all bulks are composed of 1 packet.

Similar curves can be obtained for more complex models. Figure 4 shows the behaviors of the average buffer occupancy (21) and its standard deviation (22) for a Poisson discrete probability density

$$\{g_{k,s}\} = \left\{ \frac{a^k e^{-a}}{k!} \right\}\,, \tag{25}$$

($a > 0$ is a parameter), whose z-transform is

$$G_s(z) = e^{-a(1-z^{-1})}\,. \tag{26}$$

where $a = 3$, $G'_s(1) = -3$ and $G''_s(1) = 15$.

Figures 3 and 4 show similar behaviors with respect to the parameters q and λ_s, which have a heavy impact in the system performance: the higher the values of q and λ_s, the larger the average buffer occupancy and its standard deviation. It is important to remind that q represents the level of the epidemic routing and λ_s is the average rate of bulk generation inside the node.

VI. Related Literature

An elegant model was proposed in [5] to analyze the delivery delay and its relative trade-offs with energy consumption and buffer requirements in the so-called (p, q)-*epidemic routing*. In [5] p and q represent, respectively, the probability that a node accepts a packet copy from another node when none of them is the source and the probability that a node accepts a packet copy from the packet source node. With a proper tuning of the values of p and q, (p, q)-epidemic routing models flooding, randomized flooding, or two-hops forwarding. The model is based on a continuous-time Markov chain, in which the state represents the number of copies of a specific packet in the system.

In [6], the authors developed a mathematical framework based on a Markov chain to get insights into the global congestion behavior. Their analysis is greatly simplified by replacing some random variables in the model with their expected values.

Differently from [5] and [6], in this letter we have focused on the behavior of a network single node, estimating both the discrete probability density of the size of its l-queue and the exogenous component $\{g_{k,e}\}$ of $\{g_k\}$.

VII. Conclusions

We have derived a relationship between the discrete probability densities of bulk and queue sizes, which, under a fixed bulk epidemic forwarding strategy and fixed rates, depends only on the traffic generated by single nodes towards a specific destination. This allows to compute the average buffer occupancy and its standard deviation for a specific queue that contains traffic for a given destination within a node, knowing only the discrete probability density of the size of the bulks generated by that node towards the given destination. This has immediate practical advantages, e.g., in congestion control. The model and the analysis can be extended to the case of different classes of nodes, each one associated with its own bulk generation rate.

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